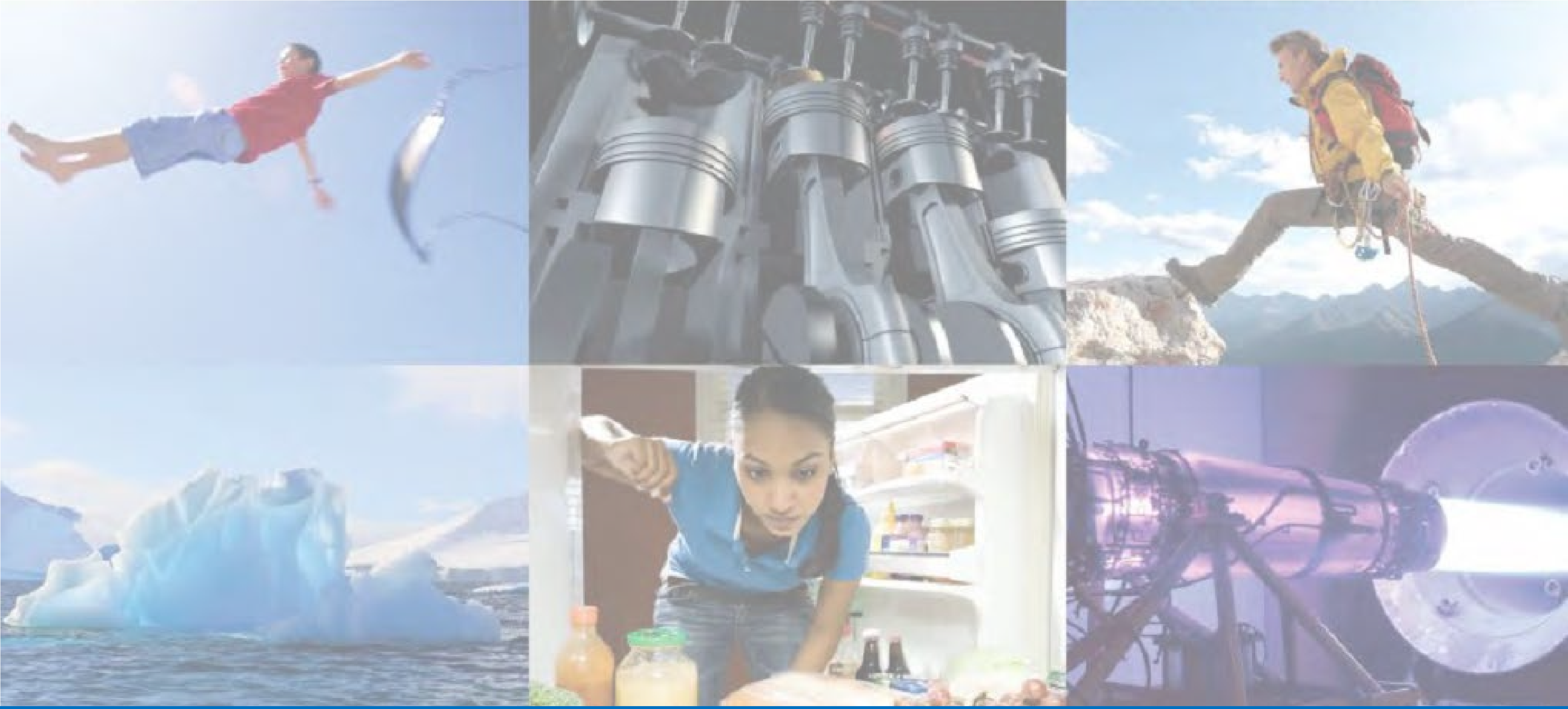


CONTROL VOLUME ANALYSIS USING ENERGY

By Ertanto Vetra

Outlines

- Mass Balance
- Energy Balance
- Steady State and Transient Analysis
- Applications

Conservation of mass

- Conservation of mass is one of the most fundamental principles in nature
- Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process
- However, mass m and energy E can be converted to each other according to the well-known formula proposed by Albert Einstein (1879–1955)

$$E = mc^2$$

Closed System vs. Control Volumes

- For **closed systems**, the conservation of mass principle is implicitly used by requiring that the mass of the system remain constant during a process.
- For **control volumes**, however, mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

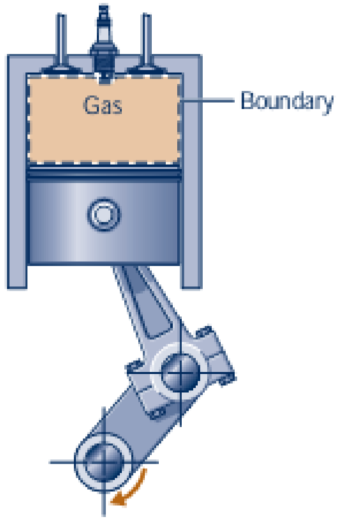


Fig. 1.1 Closed system: A gas in a piston–cylinder assembly.

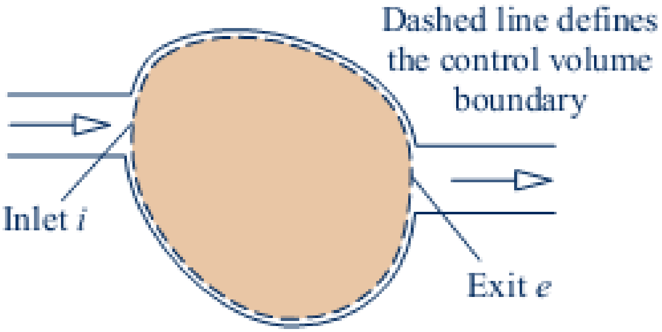


Fig. 4.1 One-inlet, one-exit control volume.

Conservation of Mass Principle

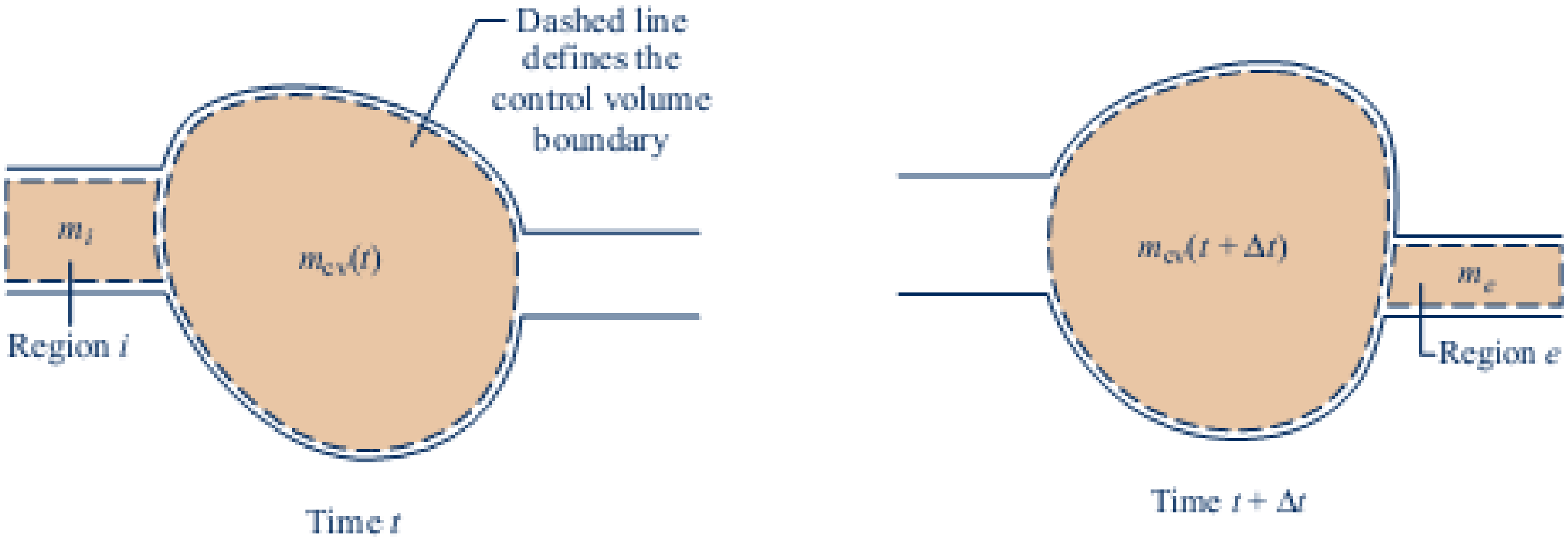
- The net mass transfer from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt

$$\begin{bmatrix} \text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within the} \\ \text{control volume } \textit{at time t} \end{bmatrix} = \begin{bmatrix} \text{time } \textit{rate} \text{ of flow of} \\ \text{mass } \textit{in} \text{ across} \\ \text{inlet } \textit{i at time t} \end{bmatrix} - \begin{bmatrix} \text{time } \textit{rate} \text{ of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } \textit{e at time t} \end{bmatrix}$$

mass rate balance for control volumes with several inlets and exits (Equation 4.2)

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

Developing the Control Volume Mass Balance



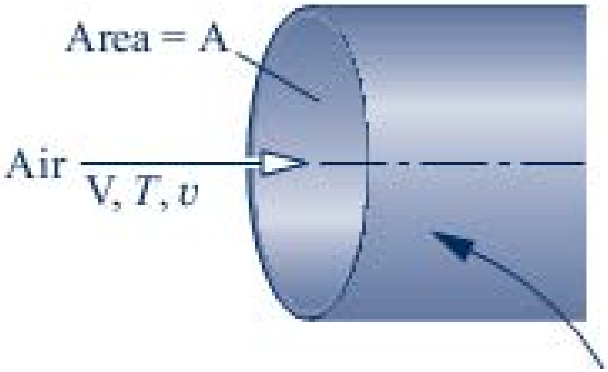
$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

Forms of Mass Balance

- In many cases, however, it is convenient to apply the mass balance in forms suited to particular objectives. Some alternative forms are:
  - One-Dimensional Flow Form of the Mass Rate Balance
  - Steady-State Form of the Mass Rate Balance
  - Integral Form of the Mass Rate Balance (not discussed)

One-Dimensional Flow Form of the Mass Rate Balance

- flow is said to be one-dimensional:
  - – The flow is <u>**normal**</u> to the boundary at locations where mass enters or exits the control volume.
  - – All intensive properties, including velocity and density, are uniform with position over each inlet or exit area

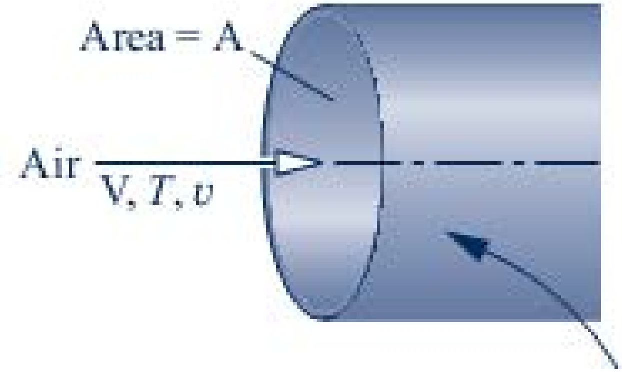


$$\dot{m} = \rho A V \qquad \text{(one-dimensional flow)}$$

Or in terms of specific volume:

$$\dot{m} = \frac{AV}{v} \qquad \text{(one-dimensional flow)}$$

One-Dimensional Flow Form of the Mass Rate Balance



$$\dot{m} = \frac{AV}{v} \qquad \text{(one-dimensional flow)}$$

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

became

$$\frac{dm_{cv}}{dt} = \sum_i \frac{A_i V_i}{v_i} - \sum_e \frac{A_e V_e}{v_e} \qquad \text{(one-dimensional flow)}$$

Steady-State Form of the Mass Rate Balance

- steady state, meaning that all properties are unchanging in time.

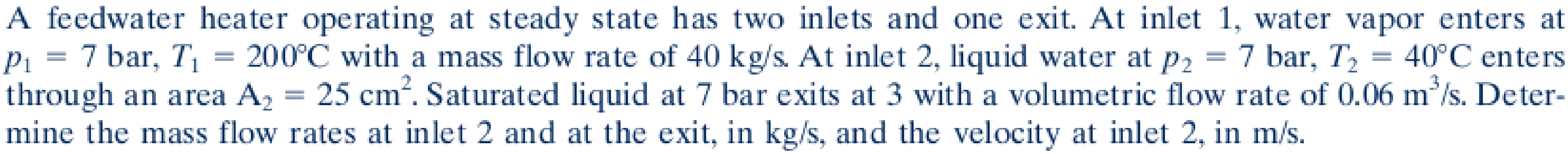
$$\frac{dm_{\text{cv}}}{dt} = \mathbf{0}$$

Equation 4.2 Reduces to:

$$\underset{(\text{mass rate in})}{\sum_i \dot{m}_i} = \underset{(\text{mass rate out})}{\sum_e \dot{m}_e}$$

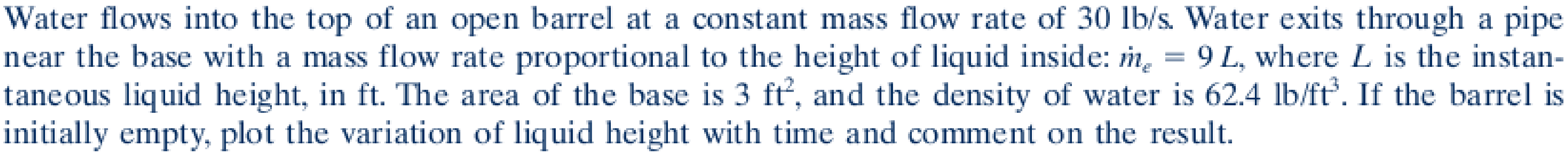
Application of Mass Balance

Example 4.1 Applying the Mass Rate Balance to a Feed water Heater at Steady State



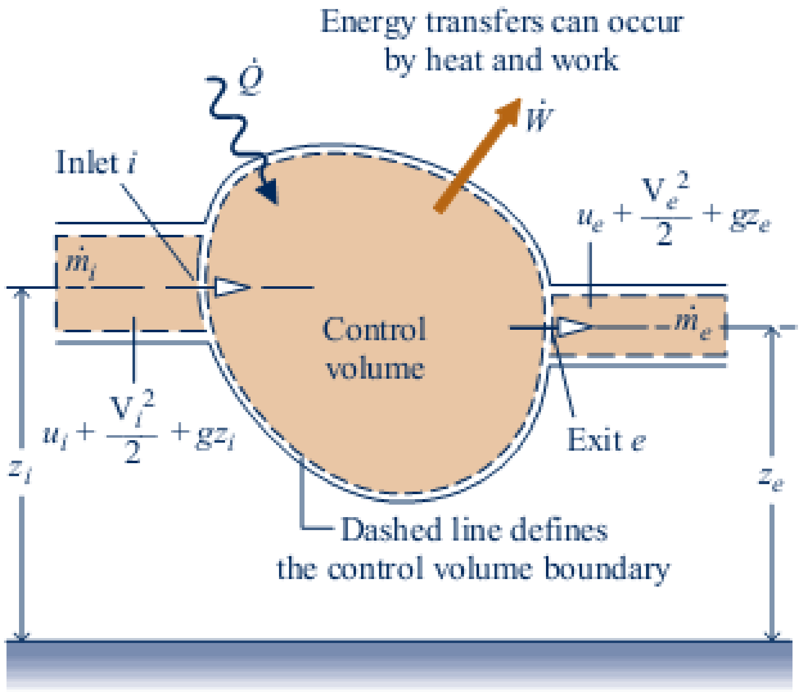
A feedwater heater operating at steady state has two inlets and one exit. At inlet 1, water vapor enters at $p_1 = 7$ bar, $T_1 = 200°\text{C}$ with a mass flow rate of 40 kg/s. At inlet 2, liquid water at $p_2 = 7$ bar, $T_2 = 40°\text{C}$ enters through an area $A_2 = 25\ \text{cm}^2$. Saturated liquid at 7 bar exits at 3 with a volumetric flow rate of $0.06\ \text{m}^3/\text{s}$. Determine the mass flow rates at inlet 2 and at the exit, in kg/s, and the velocity at inlet 2, in m/s.

Example 4.2 Applying the Mass Rate Balance to a Barrel Filling with Water



Water flows into the top of an open barrel at a constant mass flow rate of 30 lb/s. Water exits through a pipe near the base with a mass flow rate proportional to the height of liquid inside: $\dot{m}_e = 9L$, where L is the instantaneous liquid height, in ft. The area of the base is $3\ \text{ft}^2$, and the density of water is $62.4\ \text{lb/ft}^3$. If the barrel is initially empty, plot the variation of liquid height with time and comment on the result.

Conservation of Energy


Energy transfers can occur by heat and work

Dashed line defines the control volume boundary

$$\begin{bmatrix}\text{time } \textit{rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume} \\ \textit{at time t}\end{bmatrix} = \begin{bmatrix}\text{net } \textit{rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat transfer} \\ \textit{at time t}\end{bmatrix} + \begin{bmatrix}\textit{net} \text{ rate at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work } \textit{at} \\ \textit{time t}\end{bmatrix} + \begin{bmatrix}\textit{net} \text{ rate of energy} \\ \text{transfer } \textit{into} \text{ the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow}\end{bmatrix}$$

Conservation of Energy

$$\begin{bmatrix}\text{time } \textit{rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume} \\ \textit{at time t}\end{bmatrix} = \begin{bmatrix}\text{net } \textit{rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat transfer} \\ \textit{at time t}\end{bmatrix} + \begin{bmatrix}\textit{net} \text{ rate at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work } \textit{at} \\ \textit{time t}\end{bmatrix} + \begin{bmatrix}\textit{net} \text{ rate of energy} \\ \text{transfer } \textit{into} \text{ the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow}\end{bmatrix}$$

$$\frac{dE_{cv}}{dt} = \dot{Q} + \dot{W} + \underline{\dot{m}_i\left(u_i + \frac{V_i^2}{2} + gz_i\right)} - \underline{\dot{m}_e\left(u_e + \frac{V_e^2}{2} + gz_e\right)}$$

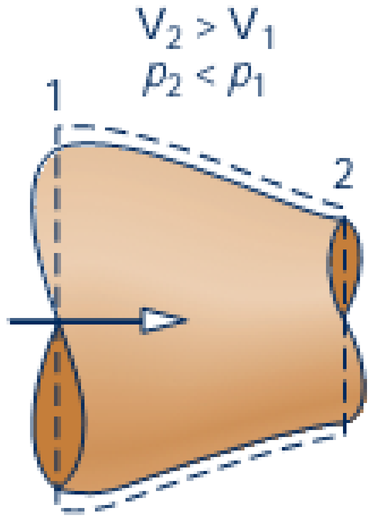
One-Dimensional Flow Form of the Control Volume Energy Rate Balance

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} + \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e \right)$$

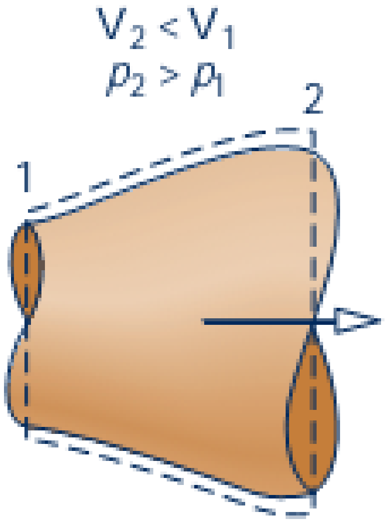
This is the main equation for this chapter, subsequent applications will be derived from this equation

CONTROL VOLUME ANALYSIS APPLICATIONS

Nozzles & Diffusers



Nozzle



Diffuser

Velocity increases

Velocity decreases

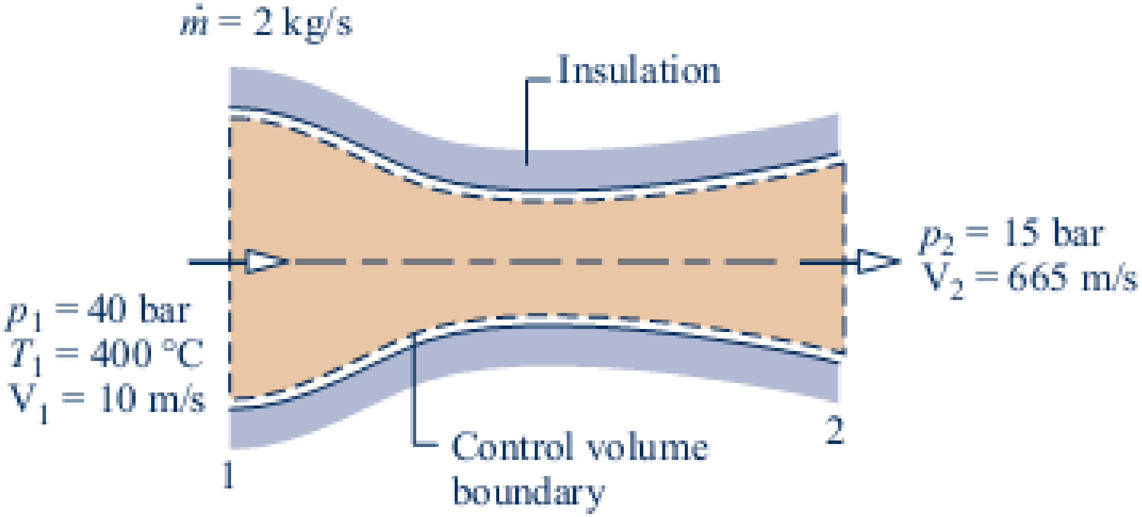
$$\boxed{0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)}$$

Assumptions:

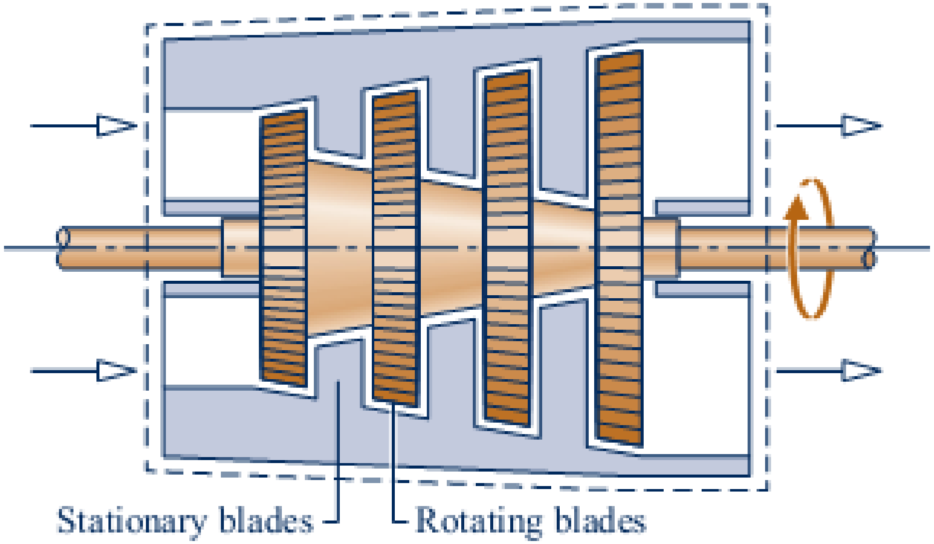
- Steady State
- Negligible heat transfer
- No change in potential energy

Example

- Steam enters a converging–diverging nozzle operating at steady state with P_1 = 40 bar, T_1 = 400 °C, and a velocity of 10 m/s. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit, P_2 = 15 bar, and the velocity is 665 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle, in m^2.



Turbines



$$\dot{W}_{cv} = - \dot{m}(h_1 - h_2)$$

Assumptions:

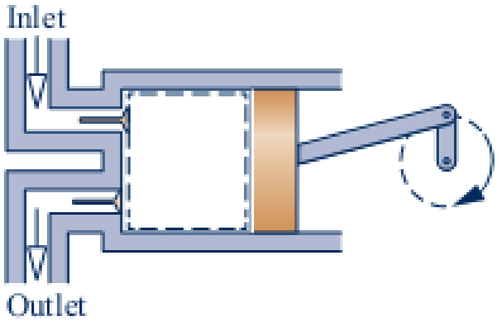
- Steady State
- Negligible heat transfer
- No change in potential energy
- Negligible change in kinetic energy

Example

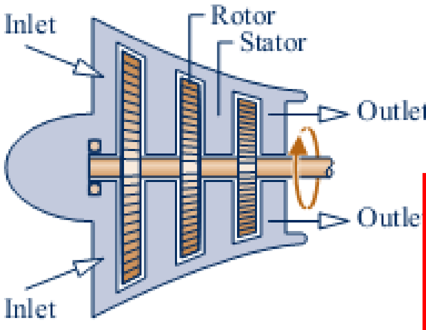
- Steam enters a turbine operating at steady state with a mass flow rate of 4600 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 60 bar, the temperature is 400°C, and the velocity is 10 m/s. At the exit, the pressure is 0.1 bar, the quality is 0.9 (90%), and the velocity is 30 m/s. Calculate the rate of heat transfer between the turbine and surroundings, in kW.

Note that in this example the kinetic energy change and heat transfer is calculated. This is to show the magnitude difference between those with the total enthalpy change. Based on the result, the previous assumption to drop the heat transfer and kinetic energy change is usually justified

Compressor & Pumps

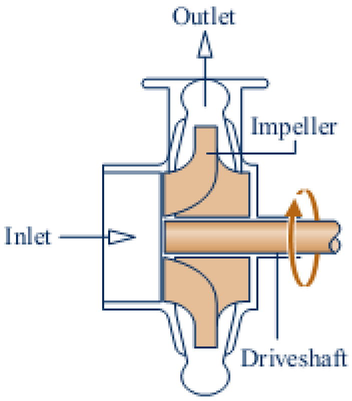


(a) Reciprocating

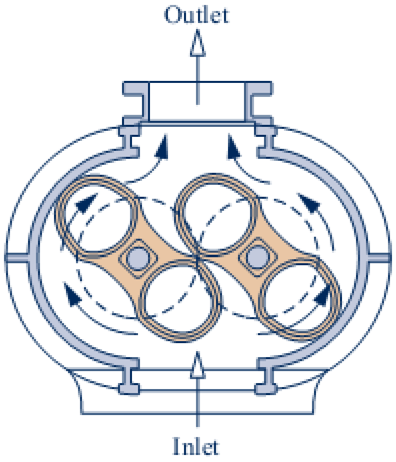


(b) Axial flow

$$\dot{W}_{cv} = - \dot{m}(h_1 - h_2)$$



(c) Centrifugal



(d) Roots type

For pumps, heat transfer is generally a secondary effect, but the kinetic and potential energy terms of Eq. 4.20a may be significant depending on the application

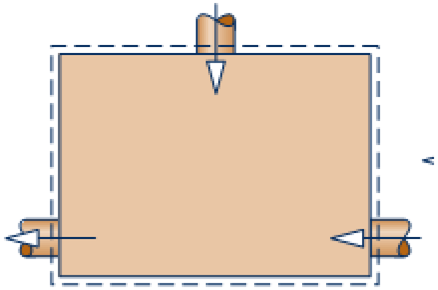
Example (Calculating Compressor Power)

- Air enters a compressor operating at steady state at a pressure of 1 bar, a temperature of 290 K, and a velocity of 6 m/s through an inlet with an area of 0.1 m^2. At the exit, the pressure is 7 bar, the temperature is 450 K, and the velocity is 2 m/s. Heat transfer from the compressor to its surroundings occurs at a rate of 180 kJ/min. Employing the ideal gas model, calculate the power input to the compressor, in kW.

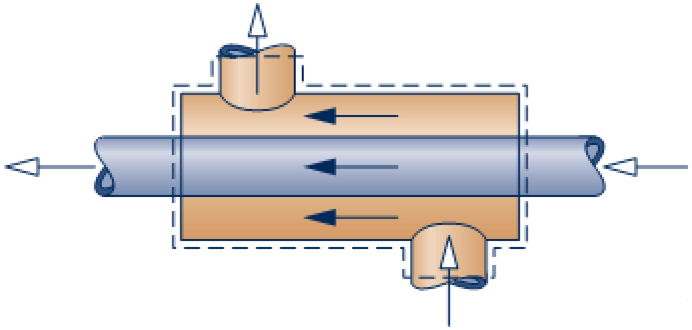
Example(analyzing a pump system)

- A pump steadily draws water from a pond at a volumetric flow rate of 0.83 m^3/min through a pipe having a 12-cm diameter inlet. The water is delivered through a hose terminated by a converging nozzle. The nozzle exit has a diameter of 3 cm and is located 10 m above the pipe inlet. Water enters at 20 ° C, 1 atm and exits with no significant change in temperature or pressure. The magnitude of the rate of heat transfer from the pump to the surroundings is 5% of the power input. The acceleration of gravity is 9.81 m/s2. Determine :
  a. the velocity of the water at the inlet and exit, each in m/s, and
  b. the power required by the pump, in kW.

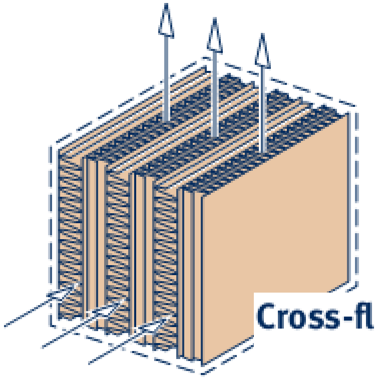
Heat Exchangers



(*a*) Direct contact heat exchanger.



Tube-within-a-tube parallel flow heat exchanger.



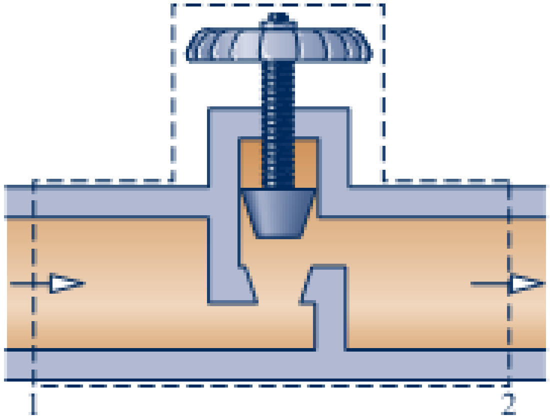
Cross-flow heat exchanger.

$$\boxed{0 = \dot{Q}_{\text{cv}} + \sum_{i} \dot{m}_i h_i - \sum_{e} \dot{m}_e h_e}$$

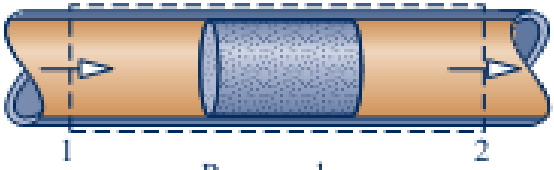
Example(Evaluating Performance of a Power Plant Condenser)

- Steam enters the condenser of a vapor power plant at 0.1 bar with a quality of 0.95 and condensate exits at 0.1 bar and 45 ° C. Cooling water enters the condenser in a separate stream as a liquid at 20 ° C and exits as a liquid at 35 ° C with no change in pressure. Heat transfer from the outside of the condenser and changes in the kinetic and potential energies of the flowing streams can be ignored. For steady-state operation, determine:
  a. the ratio of the mass flow rate of the cooling water to the mass flow rate of the condensing steam.
  b. the rate of energy transfer from the condensing steam to the cooling water, in kJ per kg of steam passing through the condenser.

Throttling Devices



Partially open valve



Porous plug

$$0 = (h_1 - h_2) + \boxed{\frac{V_1^2 - V_2^2}{2}}$$

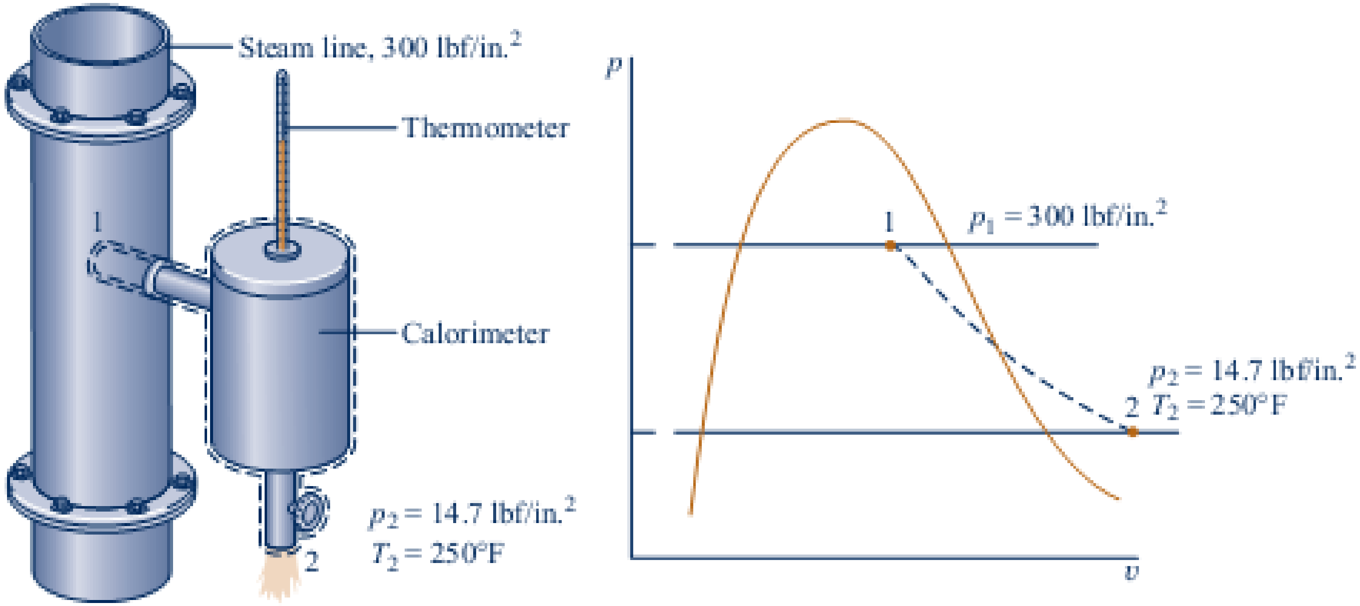
neglected

Throttling Process = Isenthalpic (constant enthalpy)

$$h_2 = h_1 \qquad (p_2 < p_1)$$

Example (Using a Throttling Calorimeter to Determine Quality)

- A supply line carries a two-phase liquid–vapor mixture of steam at 300 lbf/in². A small fraction of the flow in the line is diverted through a throttling calorimeter and exhausted to the atmosphere at 14.7 lbf/in². The temperature of the exhaust steam is measured as 250 ° F. Determine the quality of the steam in the supply line.



System Integration

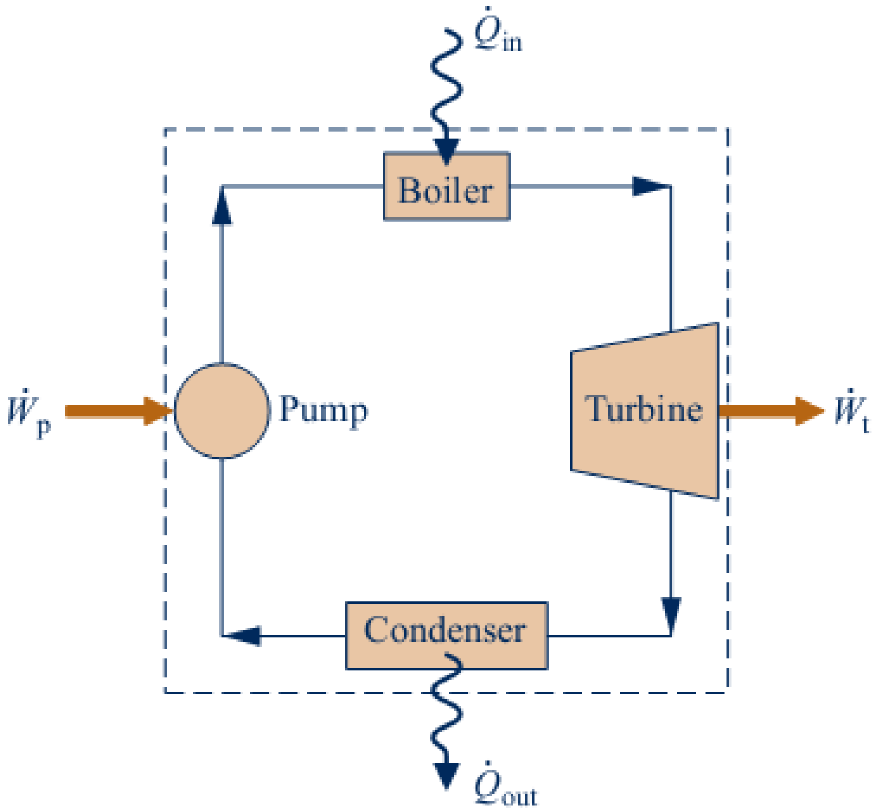


Fig. 4.16 **Simple vapor power plant.**

- We have studied several types of components selected from those commonly seen in practice
- These components are usually encountered in combination, rather than individually.
- Engineers often must creatively combine components to achieve some overall objective, subject to constraints such as minimum total cost.
- This important engineering activity is called **system integration**

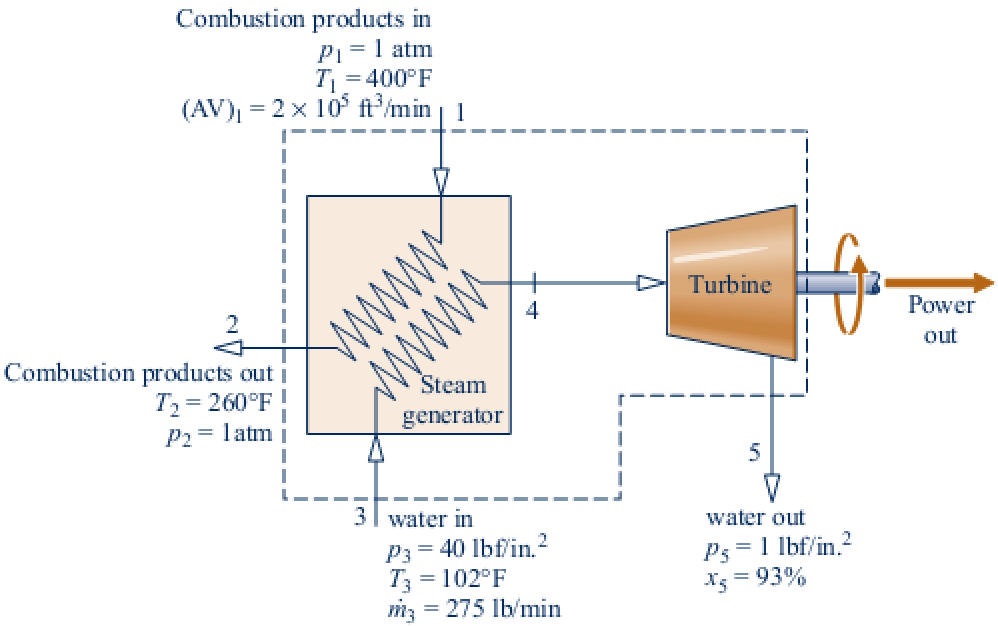
Example (Evaluating Performance of a Waste Heat Recovery System)

An industrial process discharges 2 x 10^5 ft^3/min of gaseous combustion products at 400 °F, 1 atm. As shown in Fig. E4.10, a proposed system for utilizing the combustion products combines a heat-recovery steam generator with a turbine. At steady state, combustion products exit the steam generator at 260 ° F, 1 atm and a separate stream of water enters at 40 lbf/in^2 , 102 ° F with a mass flow rate of 275 lb/min. At the exit of the turbine, the pressure is 1 lbf/in^2 and the quality is 93%. Heat transfer from the outer surfaces of the steam generator and turbine can be ignored, as can the changes in kinetic and potential energies of the flowing streams. There is no significant pressure drop for the water flowing through the steam generator. The combustion products can be modeled as air as an ideal gas.

a. Determine the power developed by the turbine, in Btu/min.
b. Determine the turbine inlet temperature, in ° F.
c. Evaluating the power developed at $0.08 per kW ? h, determine the value of the power, in $/year, for 8000 hours of operation annually.

Example (Evaluating Performance of a Waste Heat Recovery System)

Schematic and Given Data:



Engineering Model:

1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer is negligible, and changes in kinetic and potential energy can be ignored.
3. There is no pressure drop for water flowing through the steam generator.
4. The combustion products are modeled as air as an ideal gas.