ENTROPY

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Entropy

- In Chap. 6, we introduced the second law of thermodynamics and applied it to cycles and cyclic devices.

- The first law of thermodynamics deals with the property *energy* and the conservation of it. The second law leads to the definition of a new property called entropy

Entropy

- Entropy is a somewhat **abstract property**
- It is difficult to give a physical description of it without considering the microscopic state of the system
- Unlike energy, entropy is a **non-conserved property**, and there is **NO** such thing as conservation of entropy.

Clausius Inequality

The Clausius inequality states that for any thermodynamic cycle:

$$\oint \left(\frac{\delta Q}{T} \right)_b \leq 0$$

the cyclic integral of δQ/T is always less than or equal to zero

- dQ represents the heat transfer at a part of the system boundary during a portion of the cycle,
- T is the absolute temperature at that part of the boundary.
- The subscript "b" serves as a reminder that the integrand is evaluated at the boundary of the system executing the cycle.
- The symbol "circle integral" symbol indicates that the integral is to be performed over all parts of the boundary and over the entire cycle.

Clausius Inequality

The **Clausius inequality** can be expressed equivalently as

$$\oint \left(\frac{\delta Q}{T} \right)_{b} = -\sigma_{cycle} \tag{5.13}$$

where σ_{cycle} can be interpreted as representing the "strength" of the inequality. The value of σ_{cycle} is positive when internal irreversibilities are present, zero when no internal irreversibilities are present, and can never be negative.

In summary, the nature of a cycle executed by a system is indicated by the value for σ_{cycle} as follows:

$$\sigma_{cycle} = 0 \qquad \text{no irreversibilities present within the system}$$
$$\sigma_{cycle} > 0 \qquad \text{irreversibilities present within the system} \tag{5.14}$$
$$\sigma_{cycle} < 0 \qquad \text{impossible}$$

Defining Entropy Change
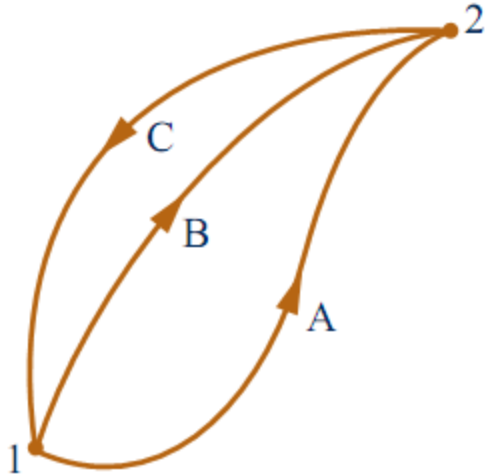
Fig. 6.1 Two internally reversible cycles.

$$\left(\int_1^2 \frac{\delta Q}{T}\right)_A + \left(\int_2^1 \frac{\delta Q}{T}\right)_C = -\sigma^{\,0}_{\text{cycle}}$$

$$\left(\int_1^2 \frac{\delta Q}{T}\right)_B + \left(\int_2^1 \frac{\delta Q}{T}\right)_C = -\sigma^{\,0}_{\text{cycle}}$$

$$\left(\int_1^2 \frac{\delta Q}{T}\right)_A = \left(\int_1^2 \frac{\delta Q}{T}\right)_B$$

It can be concluded, therefore, that the integral represents the change in some property of the system.

Selecting the symbol S to denote this property, which is called entropy, the change in entropy is given by

$$S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T}\right)_{\substack{\text{int} \\ \text{rev}}}$$

The SI unit for entropy is J/K

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Entropy Change

- Notice that we have actually defined the **change in entropy** instead of entropy itself.

- Absolute values of entropy are determined on the basis of the third law of thermodynamics, which is discussed later.

- Engineers are usually concerned with the changes in entropy.

Entropy change

The value of entropy at any state y relative to the value at the reference state x is obtained in principle from

$$S_y = S_x + \left(\int_x^y \frac{\delta Q}{T} \right)_{\substack{\text{int} \\ \text{rev}}}$$

Retrieving Entropy data - Tables

- Property Tables (Tables A-2 through A-18)
  - Vapor data
  - Saturation Data
  - Liquid Data

  The specific entropy values given in Tables A-2 through A-18 are relative to the following reference states and values. For water, the entropy of saturated liquid at 0.01°C (32.02 °F) is set to zero. For the refrigerants, the entropy of the saturated liquid at -40 °C (-40 °F) is assigned a value of zero.

Examples

- Consider two states of water. At state 1 the pressure is 3 MPa and the temperature is 500 C. At state 2, the pressure is 0.3 MPa and the specific entropy is the same as at state 1, s2=s1. Determine the temperature at state 2.

- Determine the specific entropy of Refrigerant 134a at a state where the temperature is 0 C and the specific internal energy is 138.43 kJ/kg.

- Consider two states of water. At state 1, T1 = 240 °C, p1 = 0.10 MPa. The specific enthalpy and quality are required at state 2, where p2 = 0.01 MPa and s2 = s1.

Example

- A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.

- A piston–cylinder device initially contains 3 lbm of liquid water at 20 psia and 70°F. The water is now heated at constant pressure by the addition of 3450 Btu of heat. Determine the entropy change of the water during this process.

Isentropic Process

- the entropy of a fixed mass can be changed by
  1) heat transfer, and
  2) irreversibilities
- the entropy of a fixed mass does not change during a process that is
  1) adiabatic
  2) internally reversible
- A process during which the entropy remains constant is called an **isentropic process**
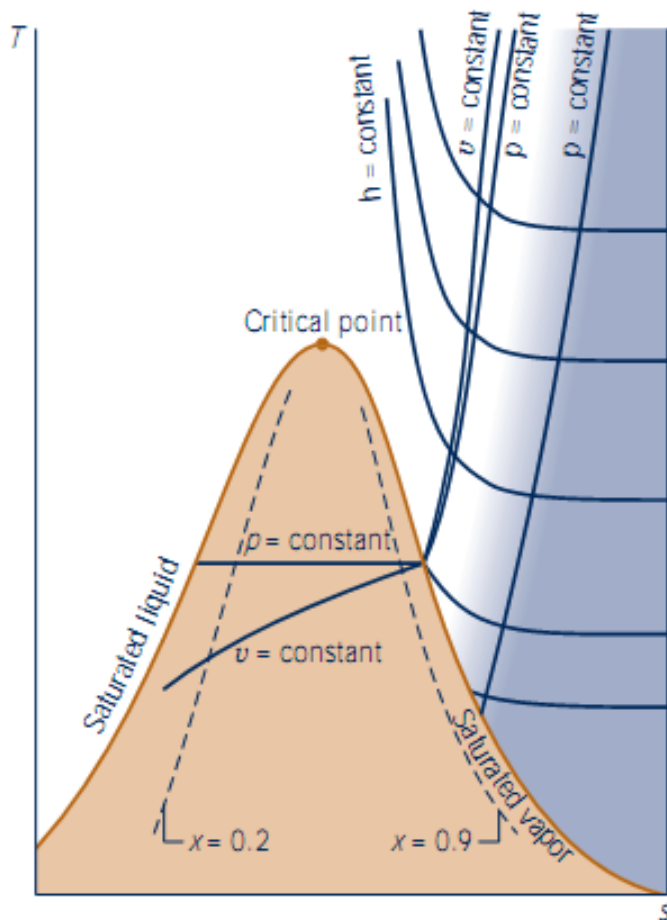
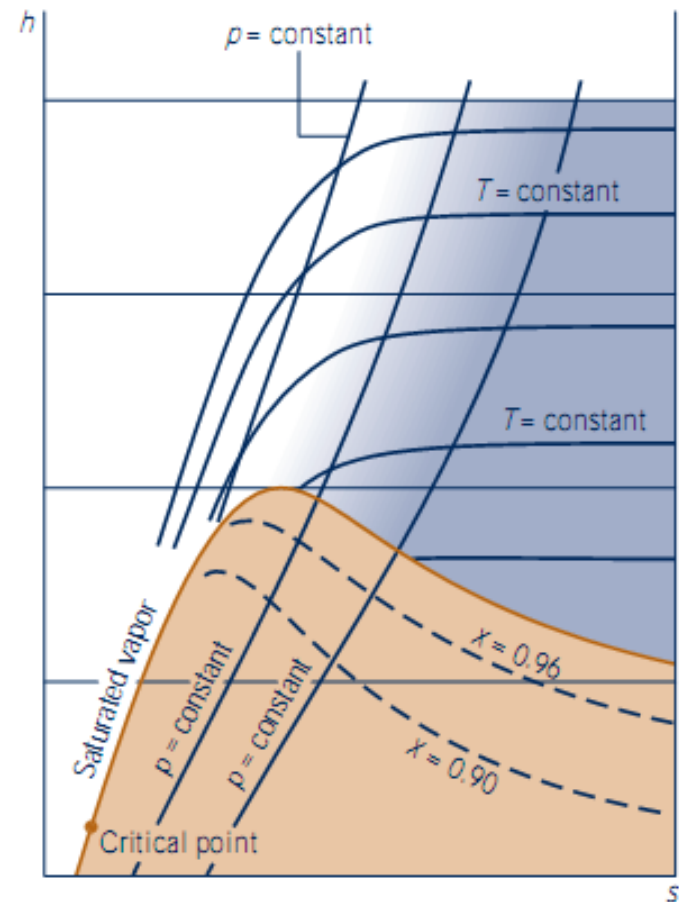*Isentropic process:* $\Delta s = 0$ or $s_2 = s_1$

Example

- Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.
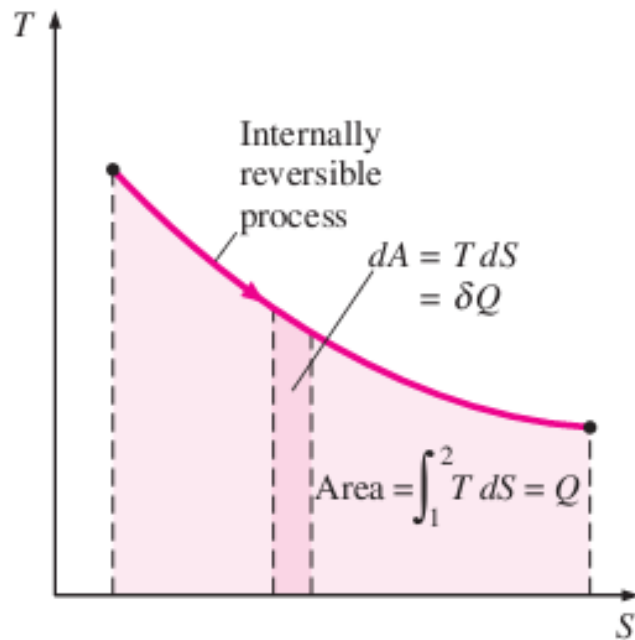
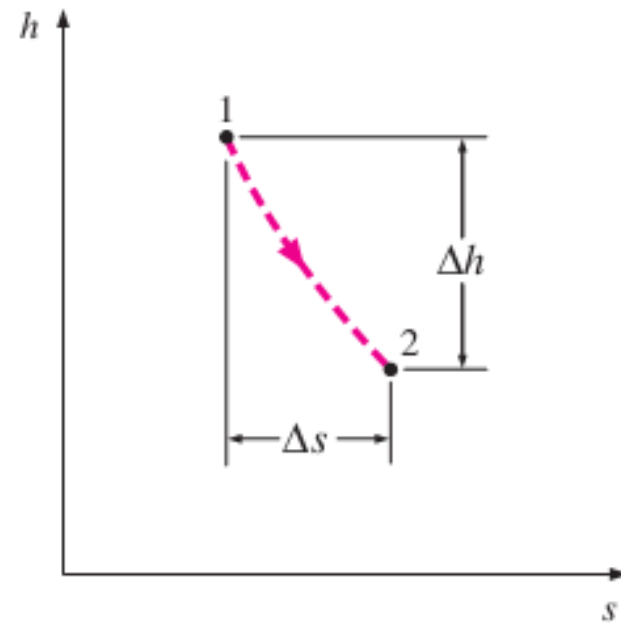## T-S Diagram (Fig. A-7)

H-S/Mollier Diagram (Fig. A-8)

FIGURE 7–16

On a *T-S* diagram, the area under the process curve represents the heat transfer for internally reversible processes.

FIGURE 7–18

For adiabatic steady-flow devices, the vertical distance Δh on an *h-s* diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

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What is entropy?

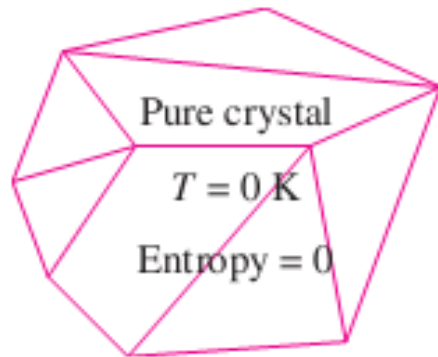
Entropy,
kJ/kg · K

GAS

LIQUID

SOLID
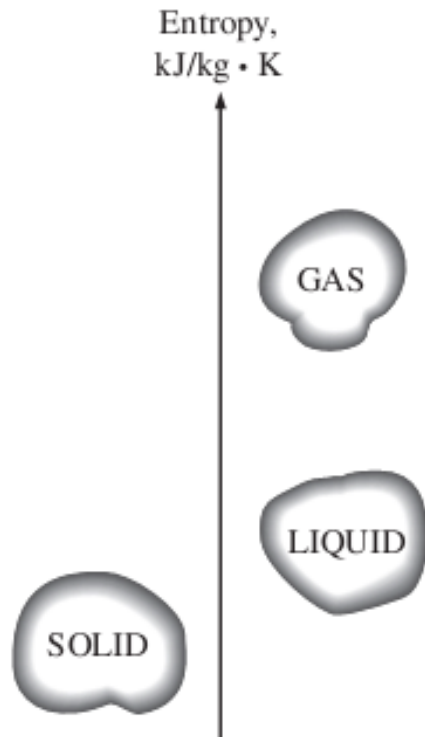
FIGURE 7–20

The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

- Entropy can be viewed as a measure of molecular disorder, or molecular randomness

- As a system becomes more disordered, the positions of the molecules become less predictable and the entropy increases

What is entropy?

Pure crystal

$T = 0 \text{ K}$

Entropy = 0

FIGURE 7–21

A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (the third law of thermodynamics).

- The molecules of a substance in solid phase continually oscillate, creating an uncertainty about their position.
- These oscillations, however, fade as the temperature is decreased, and the molecules supposedly become motionless at absolute zero.
- This represents a state of ultimate molecular order (and minimum energy).
- Therefore, the entropy of a pure crystalline substance at absolute zero temperature is zero since there is no uncertainty about the state of the molecules at that instant (Fig. 7–21).
- This statement is known as the third law of thermodynamics

*The third law of thermodynamics provides an absolute reference point for the determination of entropy. The entropy determined relative to this point is called **absolute entropy***

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Entropy change of Liquid and Solids

$$s_2 - s_1 = c \ln \frac{T_2}{T_1} \qquad \text{(incompressible, constant } c\text{)}$$

Isentropic: $\qquad s_2 - s_1 = c_{avg} \ln \dfrac{T_2}{T_1} = 0 \qquad \rightarrow \qquad T_2 = T_1$

the temperature of a truly incompressible substance remains constant during an isentropic process. Therefore, the isentropic process of an incompressible substance is also isothermal

Example

- Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or 82°C), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 7–1. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 Mpa:

  – using tabulated properties, and

  – approximating liquid methane as an incompressible substance. What is the error involved in the latter case?

Table for previous Example

TABLE 7–1

Properties of liquid methane

Temp., T, K	Pressure, P, MPa	Density, ρ, kg/m^3	Enthalpy, h, kJ/kg	Entropy, s, kJ/kg · K	Specific heat, c_p, kJ/kg · K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486

The entropy change of ideal gases

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T) \frac{dT}{T} - R \ln \frac{p_2}{p_1}$$

Using ideal gas tables (variable specific heat)

$$s(T_2, p_2) - s(T_1, p_1) = s°(T_2) - s°(T_1) - R \ln \frac{p_2}{p_1}$$

Using constant specific heat

$$s(T_2, v_2) - s(T_1, v_1) = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Example

- Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using:
    - (a) property values from the air table and
    - (b) average specific heats.

Isentropic Process – Ideal Gas

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

VALID FOR
*ideal gas
*isentropic process
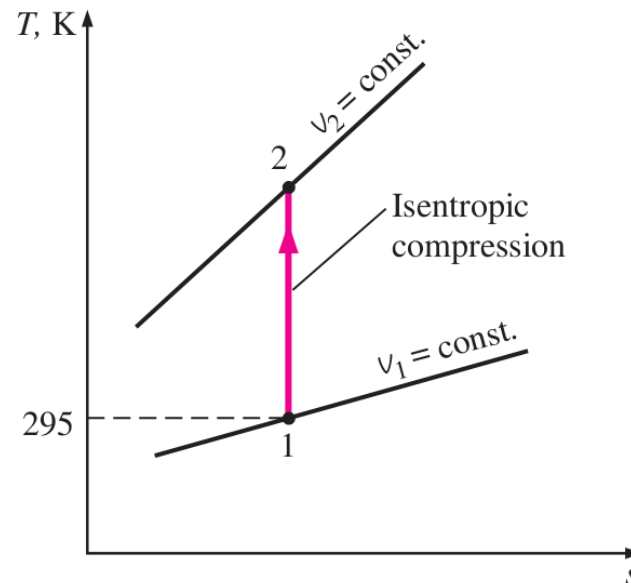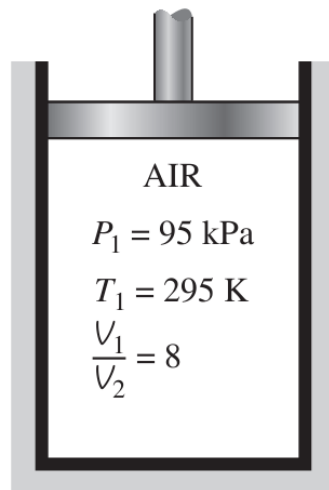*constant specific heats

Special case for Air (Table A-22)

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}} \qquad (s_1 = s_2, \text{ air only})$$

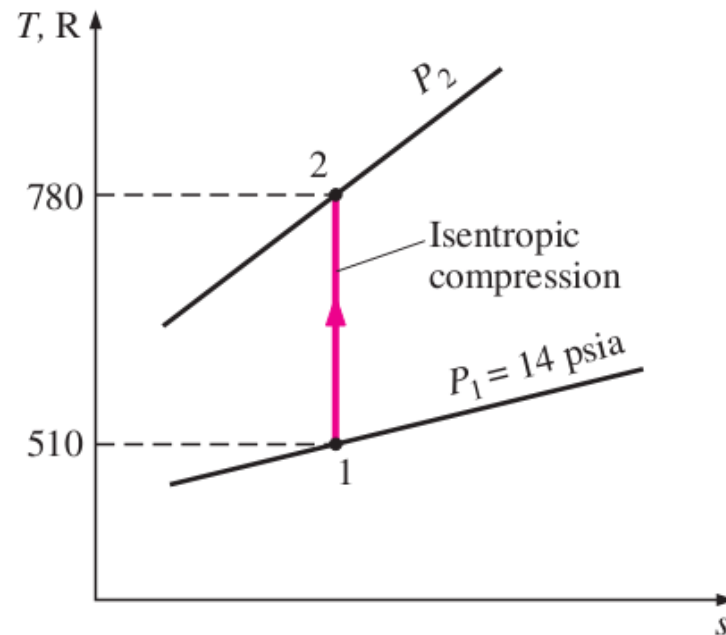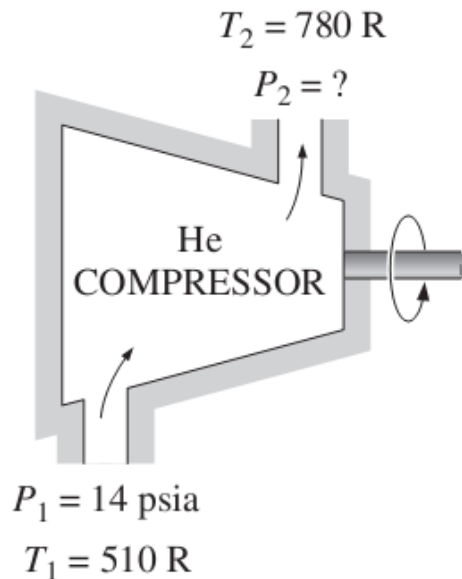$$\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}} \qquad (s_1 = s_2, \text{ air only})$$

- Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio v1/v2 of this engine is 8, determine the final temperature of the air.

AIR

$P_1 = 95 \text{ kPa}$

$T_1 = 295 \text{ K}$

$\dfrac{V_1}{V_2} = 8$

T, K

$v_2 = \text{const.}$

2

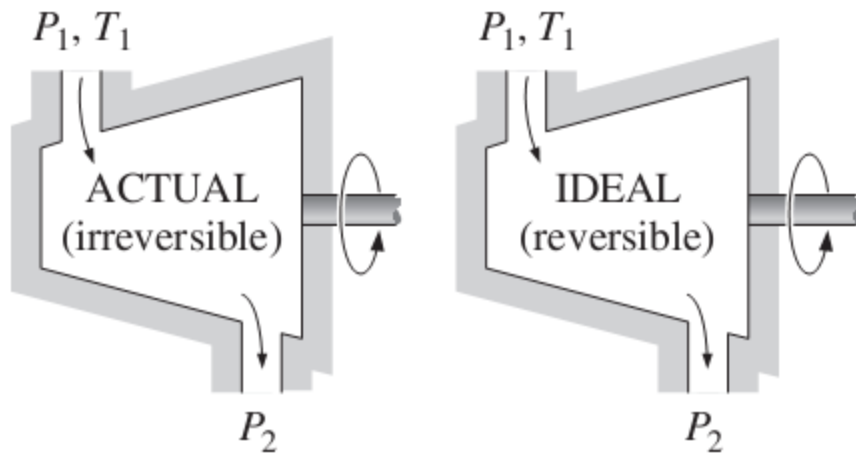Isentropic compression

$v_1 = \text{const.}$

295

1

s

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Example

- Helium gas is compressed by an adiabatic compressor from an initial state of 14 psia and 50°F to a final temperature of 320°F in a reversible manner. Determine the exit pressure of helium.

$T_2 = 780$ R
$P_2 = ?$

He COMPRESSOR

$P_1 = 14$ psia
$T_1 = 510$ R

T, R

P_2

780 --- 2

Isentropic compression

$P_1 = 14$ psia

510 --- 1

s

P_1, T_1

ACTUAL
(irreversible)

P_2

P_1, T_1

IDEAL
(reversible)

P_2

FIGURE 7–48

The isentropic process involves no irreversibilities and serves as the ideal process for adiabatic devices.

- Although some heat transfer between these devices and the surrounding medium is unavoidable, many steady-flow devices are intended to operate under adiabatic conditions.

- Therefore, the model process for these devices should be an adiabatic one.

- Furthermore, an ideal process should involve no irreversibilities

- Thus, the ideal process that can serve as a suitable model for adiabatic steady-flow devices is the isentropic process

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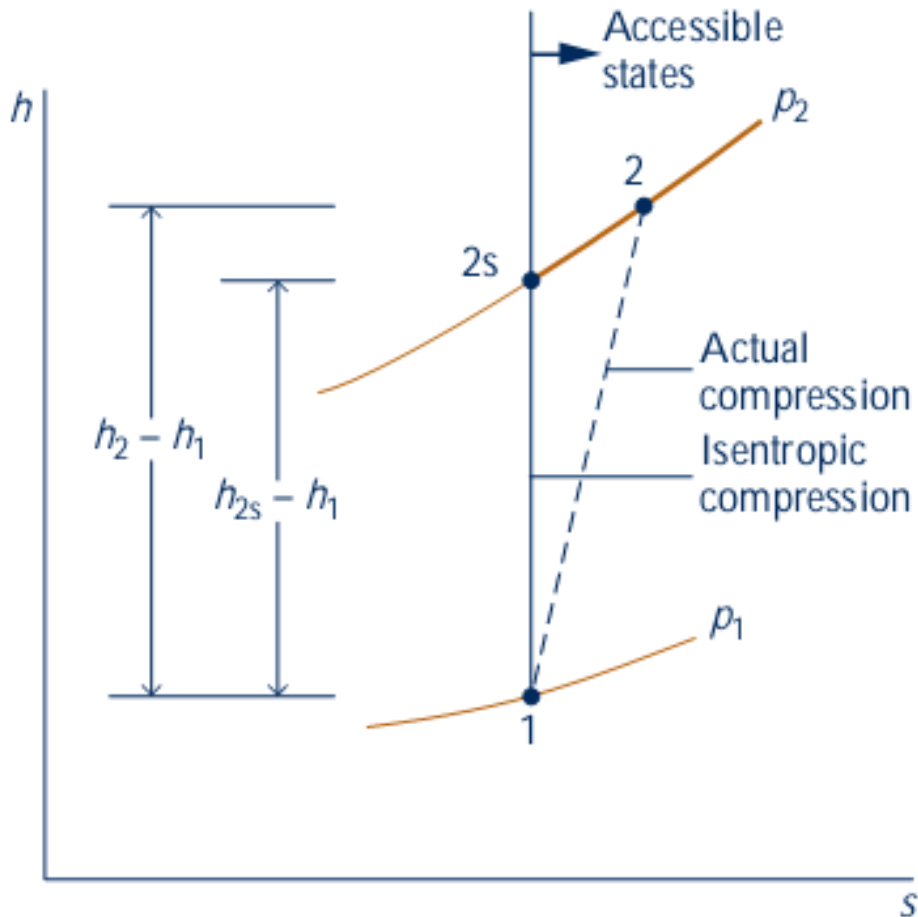
Isentropic Efficiency - Turbine
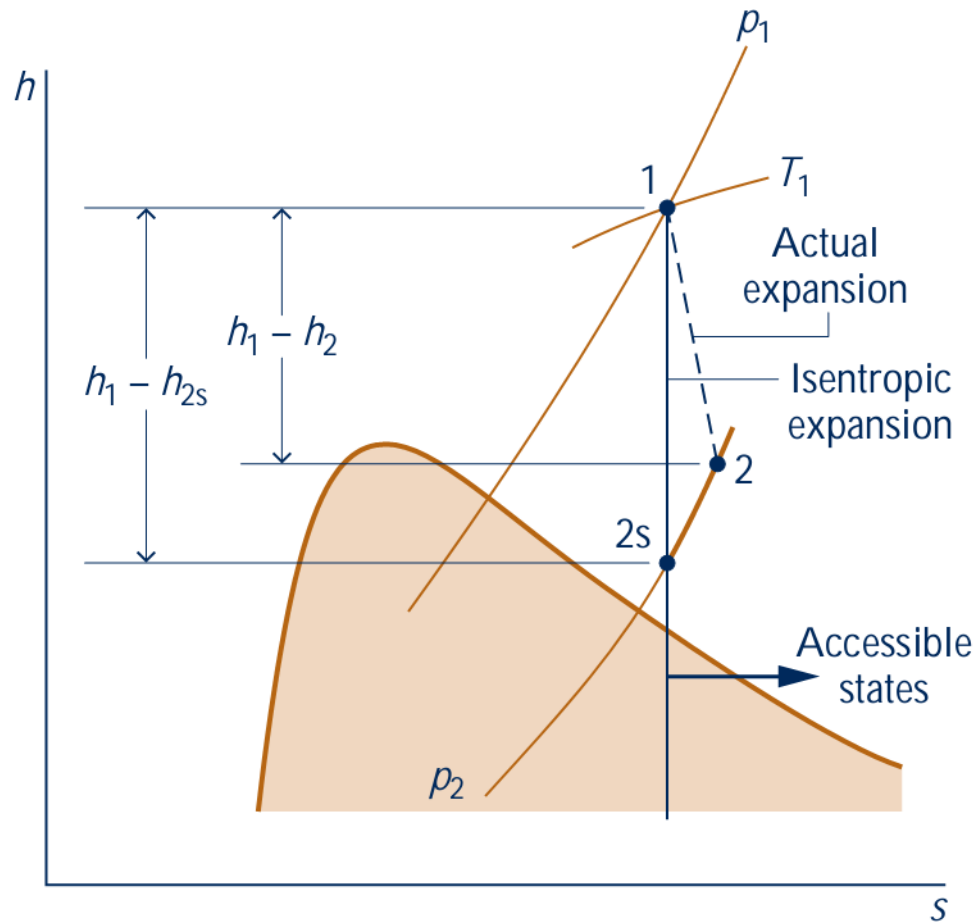
$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Example

- Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine:
    - (a) the isentropic efficiency of the turbine and
   - (b) the mass flow rate of the steam flowing through the turbine.

$$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Example

- Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine:
  - (a) the exit temperature of air and
  - (b) the required power input to the compressor.

Isentropic Efficiency – Nozzle
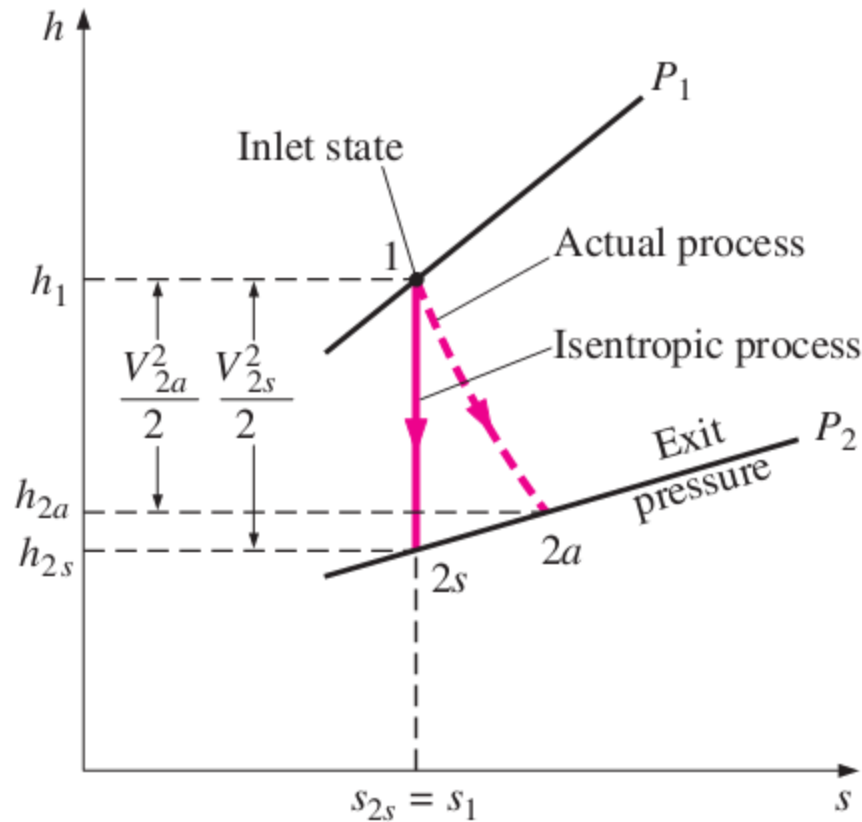
$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

$$\eta_{\text{nozzle}} = \frac{\mathrm{V}_2^2/2}{(\mathrm{V}_2^2/2)_s}$$

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 80 kPa. If the isentropic efficiency of the nozzle is 92 percent, determine:

(a) the maximum possible exit velocity,

(b) the exit temperature, and

(c) the actual exit velocity of the air. Assume constant specific heats for air.